ON FORMAL INTEGRATION OF LAGRANGE'S PLANETARY EQUATIONS OF MOTION

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ABSTRACT:

In order to solve the Lagrangian differential equations of motion the Fourier Integrals can be used. A special treatment for the so called secular terms of the disturbing function is given for that object.

1. In this paper is studied the possibility of finding a formal solution of the Lagrangian planetary differential equations of motion. It is well known that these equations can be written as follows

$$\dot{a} = -\frac{2m_1}{na}\sum i\,C\sin\theta$$

(1)

$$\dot{\Omega} = A m_1 \sum \frac{\partial C}{\partial i}\cos\theta_0 + A m_1 \sum \frac{\partial C}{\partial i}\cos\theta$$

and similar expressions for the temporal, variations of the remaining keplerian elements. The arguments θ are written in general:

$$\theta = j'l' - jl + p'\Omega' + p\Omega + q'\omega' + q\omega$$

wher the coefficients j, j' may assume zero values as well as positive (integer) values; p, p'; q, q' may have positive or negative (integral) values as well. Argument θ_0 follows by putting j=j'=0 in (2).

The integration of system (1) may be assumed, at least formally from the point of view of Fourier Integration. The fact that the time may be absent explicity in several arguments in (2) renders it necessary to pay attention to the so called secular terms of the disturbing function.

2. Let us first deal with the general case j≠0, j'≠0. We consider the general term

$$C\cos(j'l' - jl + p'\Omega' + p\Omega + q'\omega' + q\omega)$$

where

$$l = nt + \varepsilon\ ,\quad l' = n't + \varepsilon'$$

Assuming that the keplerian elements ω, ω', Ω, Ω', ε, ε', n, n', are constants we can write (3) as follows

$$C \cos (j'n't + jnt + c)$$

where $j \gtreqless j'$. Expanding the cosine, we obtain:

$$C_1' e^{ijnt} \cos j'n't + C_1'' e^{-ijnt} \cos j'n't +$$
$$+ C_2' e^{ijnt} \sin j'n't + C_2'' e^{-ijnt} \sin j'n't$$
$$i = \sqrt{-1}$$

where the C_1's contain keplerian elements and may have imaginary values in some cases.

The case in which $j \neq 0$, $j' \neq 0$ or $j=0$, $j' \neq 0$ can be solved in a little different way.

3. The main problem is logically connected to the so called secular terms. Such a term can be written, for instance

$$C \cos (p'\Omega' + p\Omega + q'\omega' + q\omega) \tag{4}$$

Let us assume that

$$C' \cos (j'n't - jnt + c) \tag{5}$$

is a term in the neiggbohood of (4), and such that $j \neq 0$, $j' \neq 0$. If in general we can assume that $C \sim C'$, we can put

$$C' = C + C'' \tag{6}$$

and then we shall get a term of the form:

$$C \cos (j'n't - jnt + c) \tag{7}$$

This new term (7) can be then combined with term (4), and we can solve the problem as in the first case. A sine term can be treated in an analogous way.

The most striking case is such one where:

$j=j' = p = p' = q = q' = 0.$

It is clear that only a coeefficient C results from the general term (2). This term can be combined with some term of its neighborhood in the development of the disturbing function, in which, for instance $j=j'=0$. We obtain in this case a term of the form

$$C'' \cos (\omega' - \omega)$$

We put here $C'' = C_1 + C$ and then the new term

$C \cos (\omega' - \omega)$, when combined with constant term C gives:

Since ω' and ω are constants by assumption, we may write

$$C_2 = C \cos \left(\frac{\omega'}{2} - \frac{\omega}{2}\right),$$ anthen we obtain a term:

$$C_1 \quad \cos \left(\frac{\omega'}{2} - \frac{\omega}{2}\right)$$

This last term can be combined with an approriate term of the disturbing function for which $j \neq 0$, $j' \neq 0$, giving rise again to the case dealt with in 1.

According to the principles settled above the development of the disturbing function will contain terms of the form:

$$f(t)\, e^{\pm ijnt}$$

Fourier Integration follows at once then, at least formally.

The range of integration may be taken for an arbitrary large interval of time t. The problem of convergence will be treated in a future paper.

REFERENCES

Papoulis, Athanasios: 1962, The Fourier Integral and its Applications.